The pro-$\mathcal{C}$ completion of the fundamental group of infinite graphs of groups

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Abstract. The profinite version of the Bass-Serre theory can be used very effectively in the study of certain abstract groups. One sees these groups as the fundamental groups of graphs of groups and through this view it is possible to apply geometric techniques to obtain algebraic results. Let $\mathcal{C}$ be an extension closed variety of finite groups, R an abstract group which is residually $\mathcal{C}$ free-by-$\mathcal{C}$ in its pro-$\mathcal{C}$ topology and denote the pro-$\mathcal{C}$ completion of R by $R_{\hat{\mathcal{C}}}$; by a result of Scott (cf. [3]), R is the fundamental group of a graph of finite groups $(\mathcal{G}, \Gamma)$. Define $\widehat{\mathcal{G}}(m)$ $(m \in \Gamma)$ to be the completion of $\mathcal{G}(m)$ with respect to this topology and take Γ as a finite graph; then it is already profinite and we define the graph of groups $(\widehat{\mathcal{G}}, \Gamma)$ in a natural way. Luis Ribes and Pavel Zalesski showed in [1] that $(\widehat{\mathcal{G}}, \Gamma)$ is injective (i.e., the restriction of the map $\nu : \widehat{\mathcal{G}} \to \Pi_1^{\mathcal{C}}(\widehat{\mathcal{G}}, \Gamma)$ to each fiber $\mathcal{G}(m)$ $(m \in \Gamma)$ is injective), the fundamental pro-$\mathcal{C}$ group $\Pi_1^{\mathcal{C}}(\widehat{\mathcal{G}}, \Gamma)$ of $(\widehat{\mathcal{G}}, \Gamma)$ is $R_{\hat{\mathcal{C}}}$ and if we assume that $\Pi_1^{abs}(m)$ is closed in the pro-$\mathcal{C}$ topology of $R = \Pi_1^{abs}(\mathcal{G}, \Gamma)$, for every $m \in \Gamma$, the standard tree $S^{abs} = S(\mathcal{G}, \Gamma)$ is canonically embedded in the $\mathcal{C}$-standard $\mathcal{C}$-tree $S = S^{\mathcal{C}}(\widehat{\mathcal{G}}, \Gamma)$ with S^{abs} dense in S. We generalise these results to the case when Γ is an infinite graph, answering Open Question 6.7.1 of [2]. It is much more subtle than the previous case, because Γ is not automatically a profinite graph. This is a joint work with Pavel A. Zalesski and it is supported by CAPES.

References

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